Name: Solution

Math 4400 Quiz 5

July 10, 2016

Instructions: You have until the end of class to complete this quiz. This quiz is two pages, and worth 20 points. Make sure to write your name at the top of the quiz. Show all of your work for full credit!

1. (10 points) Let R be a ring and let $r \in R$. Show that $(-1_R) \cdot r = -r$. In other words, show that $(-1_R) \cdot r + r = r + (-1_R) \cdot r = 0_R$.

$1_R + (-1_R) = 0_R$ (Definition of -1_R)

$\Rightarrow (1_R + -1_R) \cdot r = 0_R \cdot r$

$\Rightarrow 1_R \cdot r + (-1_R) \cdot r = 0_R \cdot r$ (Distributive prop)

$\Rightarrow r + (-1_R) \cdot r = 0_R \cdot r$ (Def. of 1_R)

$\Rightarrow r + (-1_R) \cdot r = 0_R$ (Lemma from class)

$\Rightarrow (-1_R) \cdot r + r = 0_R$ ($+$ is commutative)

2. (10 points) (a) (5 points) Find the inverse of $2+3\sqrt{3}$ in $\mathbb{Z}[\sqrt{3}]/7\mathbb{Z}[\sqrt{3}]$

$N(2+3\sqrt{3}) = 4 - 9\cdot 3 = -23 \equiv 5 \mod 7$

Guess and check: $5\cdot 3 = 15 \equiv 1 \mod 7$, so

$5^{-1} \equiv 3 \mod 7$.

$\Rightarrow (2+3\sqrt{3})^{-1} \equiv (2-3\sqrt{3})\cdot 3 \equiv 6 - 9\sqrt{3} \equiv 6+5\sqrt{3}$

(b) (5 points) Find the inverse of $2+3\sqrt{5}$ in $\mathbb{Z}[\sqrt{5}]/7\mathbb{Z}[\sqrt{5}]$

$N(2+3\sqrt{5}) = 4 - 9\cdot 5 = -41 \equiv 1 \mod 7$

$\Rightarrow (2+3\sqrt{5})^{-1} \equiv 2-3\sqrt{5} \equiv 2+4\sqrt{5}$