Name: Solutions

Math 4400 Quiz 2

June 2, 2016

Instructions: You have until the end of class to complete this quiz. This quiz is two pages, and worth 20 points. Make sure to write your name at the top of the quiz. Show all of your work for full credit!

1. (10 points) Find all integer solutions to $427x + 259y = 7$

Start with Euclidean algorithm on 427 & 259:

① $427 = 1 \cdot 259 + 168$, ② $259 = 1 \cdot 168 + 91$, ③ $168 = 1 \cdot 91 + 77$

④ $91 = 77 - 14$, ⑤ $77 = 5 \cdot 14 + 7$ $\quad 14 = 2 \cdot 7$

$77 - 5 \cdot 14 = 7$,

$77 - 5(91 - 77) = 7$ by equation ④

$\Rightarrow 6 \cdot 77 - 5 \cdot 91 = 7$

$\Rightarrow 6 \cdot (168 - 91) - 5 \cdot 91 = 7$ by ③

$\Rightarrow 6 \cdot 168 - 11 \cdot 91 = 7$

$\Rightarrow 6 \cdot 168 - 11 \cdot (259 - 168) = 7$ by ②

$\Rightarrow 17 \cdot 168 - 11 \cdot 259 = 7$

$\Rightarrow \boxed{17} \cdot 427 \boxed{-28} \cdot 259 = 7$

x_0 $\quad y_0$

$\Rightarrow \left(17 + k \cdot \frac{259}{7}, -28 - k \cdot \frac{427}{7}\right) = (17 + 37k, -28 - 61k)$

for $k \in \mathbb{Z}$,

2. (10 points) Suppose $a, b \in \mathbb{Z}$ and $a \neq 0$. Suppose also that $c \mid a$ and $c \mid b$. Show that $c \mid \gcd(a, b)$

By Bezout's lemma, $\exists r, s \in \mathbb{Z}$ such that $ar + bs = \gcd(a,b)$.

But $c|a$ and $c|b$, so $c|(ar+bs)$.

Thus $c| \gcd(a,b)$.