Math 4400 Homework 0

Due: Wednesday, May 17, 2017

Turn this assignment in at the start of class on Wednesday. The purpose of this assignment is to help people decide if they can succeed in this course, and also for me to have a sense of where everyone's skills are at. Thus, you should attempt to finish the whole thing by yourself. If you work on some of the problems with a classmate, please make a note of it on your assignment. Let me know if any of the wording is vague or confusing. Problems 1,2, and 3a should be doable, while problems 3b and 4 might be pretty hard.

1. **Linear algebra:** Let A be an $n \times n$-matrix with real entries, for some positive integer n. Suppose also that A is invertible. Show that $\det(A) \neq 0$. (Hint: if B is any other $n \times n$-matrix, then $\det(AB) = \det(A) \cdot \det(B)$.)

2. Let $i = \sqrt{-1}$. What's i^{2017}? (Hint: start by simplifying $i, i^2, i^3, i^4, i^5, \ldots$ and see if you can find a pattern!)

3. **Some Set Theory:** Let A and B be two sets. Recall the following notation:

   - "$\in$" is read "in". So "$x \in A$" is read "x is in A" or "x is an element of A".
   - $A \cap B$ denotes the *intersection* of A and B, i.e. the set of things that are in A and also in B.
   - $A \cup B$ denotes the *union* of A and B, i.e. the set of things that are in A or in B (or both).
   - $A \smallsetminus B$ denotes the *relative complement*, or *set difference* of B in A, i.e. the set of things that are in A but *not* in B.

   Now answer the following:

   (a) Let $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{4, 5, 6, 7, 8, 9, 10\}$. Find $A \cap B$, $A \cup B$, and $A \smallsetminus B$.

   (b) Now let A, B, and C be any sets. Prove the following equality:

   $$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$

   Hint: it's enough to show that

   $$(A \cup B) \cap C \supseteq (A \cap C) \cup (B \cap C)$$

   and

   $$(A \cup B) \cap C \subseteq (A \cap C) \cup (B \cap C)$$

4. Let $v \in \mathbb{R}^2$ and $w \in \mathbb{R}^2$ be two vectors. For any vector a, let $||a||$ denote the length of a. Prove the following inequality:

   $$||v + w|| \leq ||v|| + ||w||$$