Finish 3.1
Start 3.2

Q Find a matrix A s.t. $\ker(A)$ is spanned by $\begin{bmatrix}1\\1\\2\\3\end{bmatrix}$.

i.e. find A s.t. $A\vec{x}=\vec{0} \iff \vec{x}=c\begin{bmatrix}1\\1\\2\\3\end{bmatrix}$

"Given the solutions, what were the equations?"

if $\vec{x}=\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=c\begin{bmatrix}1\\1\\2\\3\end{bmatrix} \Rightarrow$

$x_1 = x_2,$
$2x_1 = x_3$
$x_4 = 3x_1$
$-x_4 = -3x_2$

$\rightarrow$
$x_1 - x_2 = 0$
$2x_1 - x_3 = 0$
$-3x_1 + x_4 = 0$

$(c=4)$ $\begin{bmatrix}4\\4\\2\cdot 4\\3\cdot 4\end{bmatrix}$

$\Updownarrow$

$$\begin{bmatrix}1 & -1 & 0 & 0\\ 2 & 0 & -1 & 0\\ -3 & 0 & 0 & 1\\ 0 & 3 & 0 & -1\end{bmatrix}\begin{bmatrix}x_1\\ \vdots\\ x_4\end{bmatrix}=\vec{0}$$

Notice: if A is any $m\times n$ matrix, then $\vec{0}_n \in \ker A$

because $A\vec{0}_n = \vec{0}_m$

$\vec{0}_n$ is the only thing in the kernel $\iff$

The system $A\vec{x}=\vec{0}_m$ has a unique solution $\iff$

pivot in each col. of A

Recall: $\text{im}(A) = \mathbb{R}^m \iff$ pivot in each row.

If A is an $n \times n$ square matrix, then

pivot in each row $\iff \text{im}(A) = \mathbb{R}^n$

$\Updownarrow$

pivot in each column $\iff \ker(A) = \{\vec{0}_n\}$

$\Updownarrow$

$\text{rref}(A) = I_n \iff A$ is invertible.

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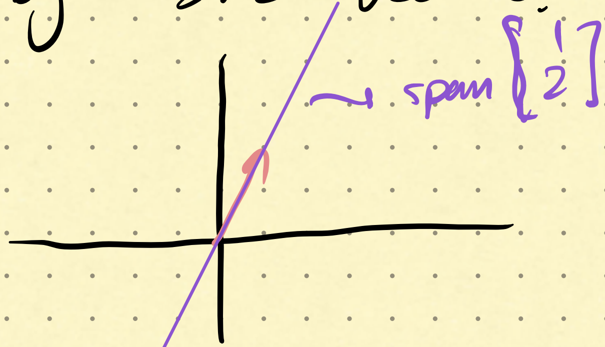
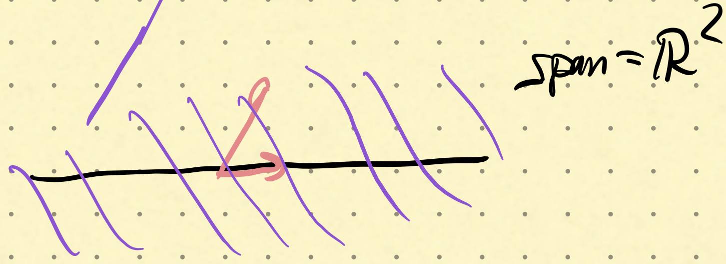
§3.2: Subspaces.

Def A <u>subspace</u> of $\mathbb{R}^n$ is a shape in $\mathbb{R}^n$ which is spanned by some vectors.

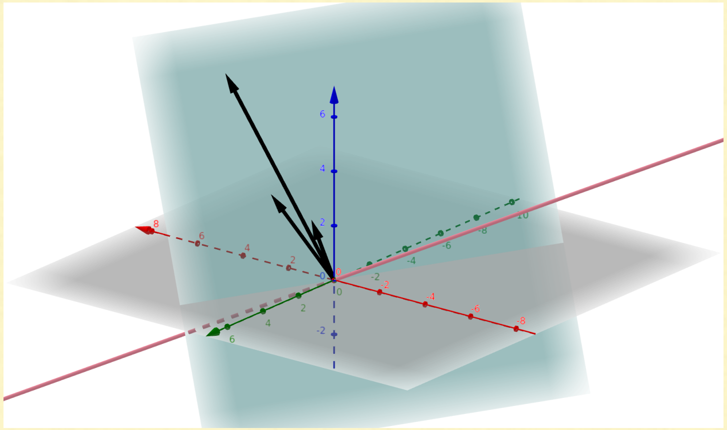
e.g. $\text{span}\left\{\begin{bmatrix}1\\2\end{bmatrix}\right\} \subseteq \mathbb{R}^2$

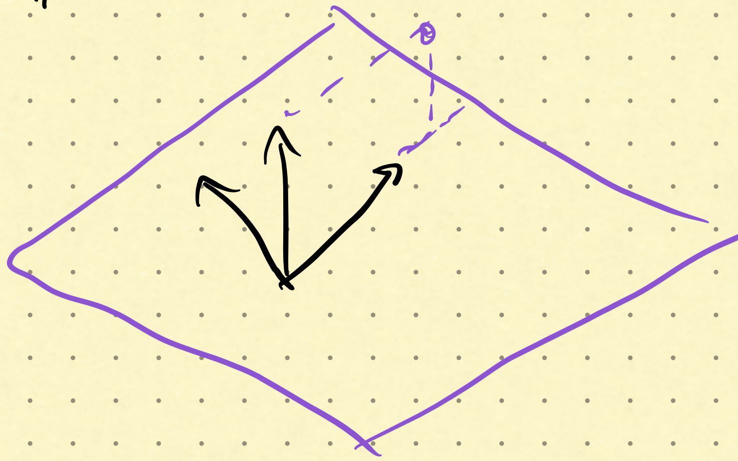


e.g. $\text{span}\left\{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}\right\} \subseteq \mathbb{R}^2$



e.g. $\text{span}\left\{ \begin{bmatrix}1\\0\\2\end{bmatrix}, \begin{bmatrix}2\\1\\3\end{bmatrix}, \begin{bmatrix}4\\1\\7\end{bmatrix} \right\} \subseteq \mathbb{R}^3$





3 vectors in one plane

their span is a plane in $\mathbb{R}^3$.

Intuition

~~FACTS~~ about subspaces:

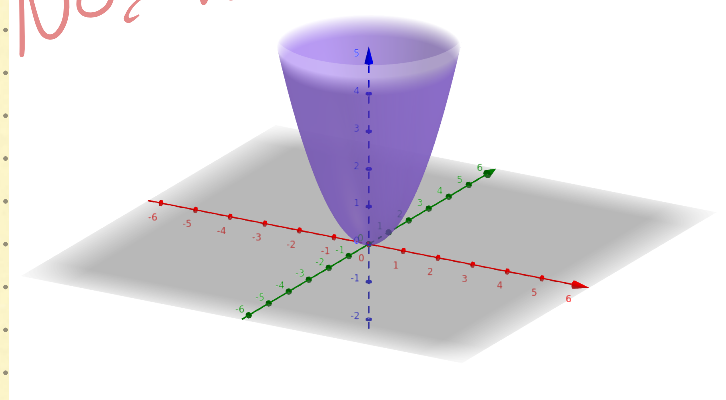
- "Flat" shapes (lines, planes, etc)
- Go on forever
- go thru the origin

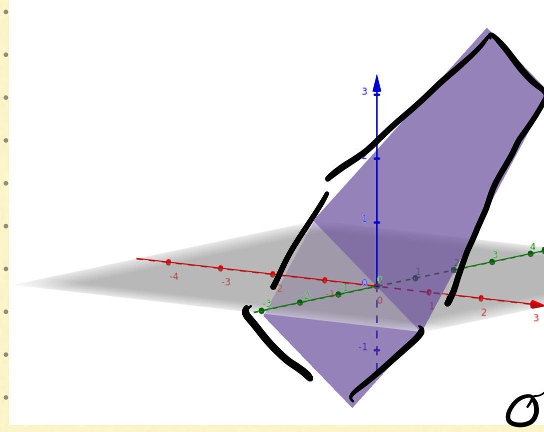
Q Why must it go thru the origin?

If $S = \text{span}\{\vec{v}_1, \dots, \vec{v}_r\}$, then $S \ni c\vec{v}_1$ for all $c \in \mathbb{R}$. In particular, $S \ni 0 \cdot \vec{v}_1 = \vec{0}$

Q Which of these are subspaces of $\mathbb{R}^3$?

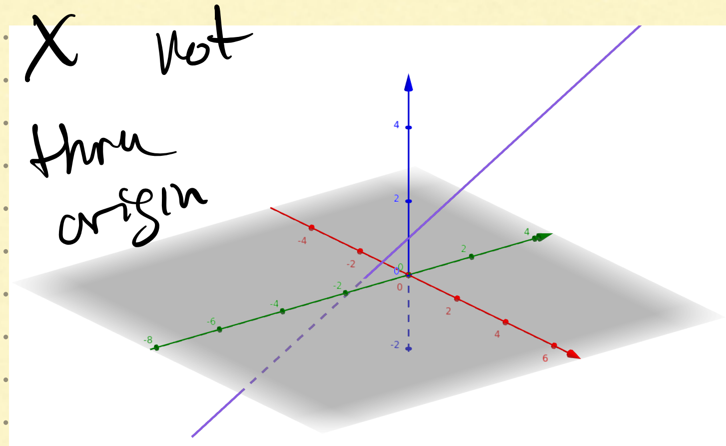
No; not flat



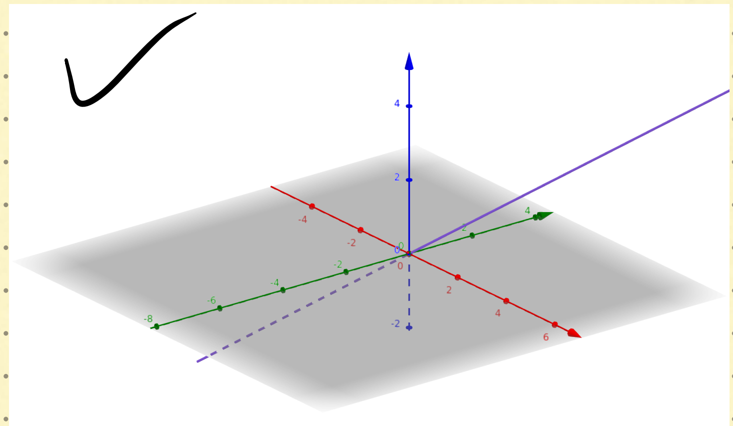


Depends on interpret. If it goes forever, ✓ otherwise, X

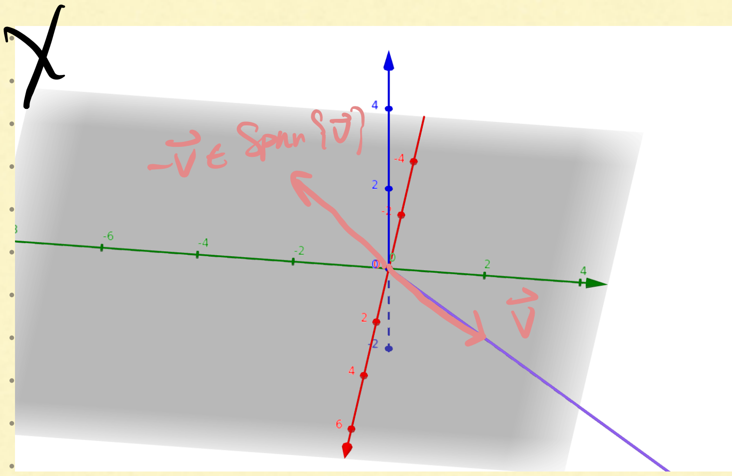
X not thru origin



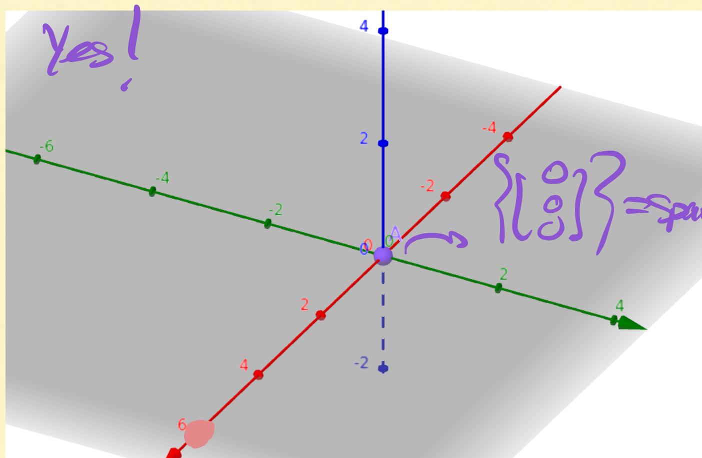
✓



X



Yes!



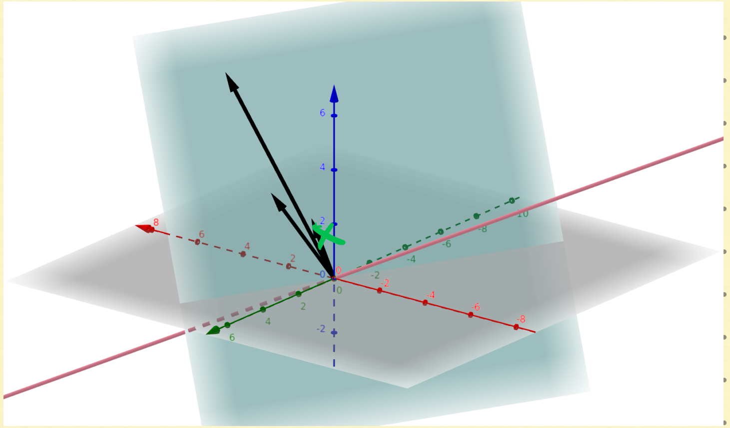
not a subspace! Doesn't go thru origin.

In $\mathbb{R}^2$, subspaces are either: origin, lines thru origin, or all $\mathbb{R}^2$

In $\mathbb{R}^3$: origin, lines thru origin, or planes thru origin, or all $\mathbb{R}^3$

etc.

Linear independence:



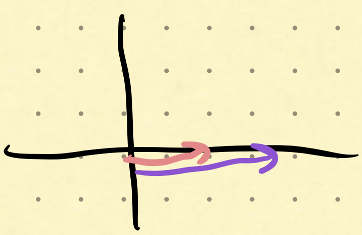
$$\text{Span}\left\{\begin{bmatrix}1\\0\\2\end{bmatrix},\begin{bmatrix}2\\1\\3\end{bmatrix},\begin{bmatrix}4\\1\\7\end{bmatrix}\right\}$$
$$\shortparallel$$
$$\text{Span}\left\{\begin{bmatrix}1\\0\\2\end{bmatrix},\begin{bmatrix}2\\1\\3\end{bmatrix}\right\}$$
$$\shortparallel$$
$$\text{Span}\left\{\begin{bmatrix}2\\1\\3\end{bmatrix},\begin{bmatrix}4\\1\\7\end{bmatrix}\right\}$$

So the vector $\begin{bmatrix}4\\1\\7\end{bmatrix}$ is redundant: adding it to our set of vectors does not change the span.
The vectors $\begin{bmatrix}1\\0\\2\end{bmatrix}, \begin{bmatrix}2\\1\\3\end{bmatrix}, \begin{bmatrix}4\\1\\7\end{bmatrix}$ are "linearly dependent":
$$\begin{bmatrix}4\\1\\7\end{bmatrix} \in \text{span}\left\{\begin{bmatrix}1\\0\\2\end{bmatrix},\begin{bmatrix}2\\1\\3\end{bmatrix}\right\}$$

<u>Def</u> A set of vectors $\vec{v}_1, \ldots, \vec{v}_r$ is called "linearly independent"

if there's no "redundancy," i.e., no vector is in the span of the others.

<u>eg</u>. $\left\{\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}2\\0\end{bmatrix}\right\}$ is lin. dependent:
$$\begin{bmatrix}2\\0\end{bmatrix} \in \text{span}\left(\begin{bmatrix}1\\0\end{bmatrix}\right)$$



$$\begin{bmatrix}1\\0\end{bmatrix} \notin \text{span}\left(\begin{bmatrix}2\\1\end{bmatrix}\right)$$

<u>eg</u>. $\left\{\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}2\\1\end{bmatrix}\right\}$, $\quad \begin{bmatrix}2\\1\end{bmatrix} \in \text{span}\left(\begin{bmatrix}1\\0\end{bmatrix}\right)$

linearly <u>independent</u>.

exit ticket:

1) Write a matrix whose kernel is a line in $\mathbb{R}^2$

2) Ask a question.