Math 214 5.3 worksheet

- 1. If A is any matrix, recall that A^T denotes the *transpose* of A. This is defined as the matrix where the columns of A^T are the rows of A, and vice-versa.
 - (a) Write down the transposes of the following matrices:

$$A = \begin{bmatrix} \pi & e \\ \sqrt{2} & 1/3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 7 \end{bmatrix}, \quad D = \begin{bmatrix} 1 \\ 3 \\ 3 \\ 7 \end{bmatrix}$$

$$A^{T} = \begin{bmatrix} \pi & e \\ \sqrt{2} & 1/3 \end{bmatrix}, \quad B^{T} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 7 \end{bmatrix}, \quad D^{T} = \begin{bmatrix} 1 & 3 & 3 & 7 \end{bmatrix}$$

(b) What is $(A^T)^T$ (the transpose of A-transpose)? In general, if E is any matrix, what is $(E^T)^T$?

$$(A^T)^T = \begin{bmatrix} \pi & e \\ 12 & 1/3 \end{bmatrix} = A$$
 In general: $(E^T)^T = E$

(c) Suppose E is a matrix with m rows and n columns. How many rows and columns does E^T have?

2. Let $\vec{v} = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 4 \\ 0 \\ 3 \end{bmatrix}$. Compute the dot product $\vec{v} \cdot \vec{w}$ and the matrix product $\vec{v}^T \vec{w}$. What do you notice?

$$\vec{v} \cdot \vec{\omega} = | \cdot 4 + 3 \cdot 4 + 2 \cdot 3 = -2$$
 $\vec{v} \cdot \vec{\omega} = [1 3 - 2] (3) = [-2]$

They're the same!

3. Let
$$A = \begin{bmatrix} 0 & 4 \\ 0 & -1 \\ 1 & 0 \end{bmatrix}$$
 and $B = \begin{bmatrix} -1 & 2 \\ 3 & 1 \\ 2 & 0 \end{bmatrix}$

(a) Find
$$A^TB$$
.

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.

$$\begin{bmatrix}
0 & 0 & 1 \\
4 & -1 & 0
\end{bmatrix}
\begin{bmatrix}
-1 & 2 \\
3 & 1 \\
2 & 0
\end{bmatrix}
=
\begin{bmatrix}
2 & 0 \\
-7 & 7
\end{bmatrix}$$

(b) Let $\vec{v_1}, \vec{v_2}$ be the columns of A and let $\vec{w_1}, \vec{w_2}$ be the columns of B. Compute all the dot products $\vec{v}_i \cdot \vec{w}_i$. What do you notice?

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} = 2$$

$$\begin{bmatrix} 4 \\ -5 \end{bmatrix} \circ \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} = -7$$

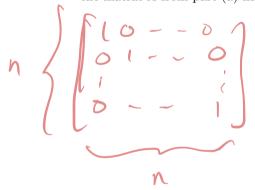
$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \circ \begin{bmatrix} 2 \\ 0 \end{bmatrix} = 0$$

(c) Let $A = [\vec{a}_1 \dots \vec{a}_r]$ and $B = [\vec{b}_1, \dots, \vec{b}_s]$, where \vec{a}_i and \vec{b}_j are all vectors in \mathbb{R}^n . Is the matrix product A^TB well-defined? (That is, do these two matrices have the correct numbers of rows/columns so that this product is defined?) If so, can you guess how the entries of A^TB relate to the dot products of the various \vec{a}_i and \vec{b}_j vectors?

4. (a) Let
$$A = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$
. Compute $A^T A$.



(b) Suppose B is an $m \times n$ -matrix with orthonormal columns. What can you say about $B^T B$? (Hint: the matrix A from part (a) has orthonormal columns!)



BTB= In, the nxn identity metrix!

(c) Suppose $\vec{u}_1, \ldots, \vec{u}_n$ is some orthonormal basis of \mathbb{R}^n . Let $Q = [\vec{u}_1 \cdots \vec{u}_n]$. In other words the columns of Q are the vectors $\vec{u}_1, \ldots, \vec{u}_n$. Is Q invertible? If so, is there a quick way to find Q^{-1} ?





(key point!)

5. Let

$$\vec{u}_1 = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \\ 0 \end{bmatrix}, \quad \vec{u}_2 = \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{bmatrix},$$

and let $V \subseteq \mathbb{R}^4$ be the subspace spanned by \vec{u}_1 and \vec{u}_2 .

(a) Let $\vec{w} = \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}^T$. Compute $\text{proj}_V(\vec{w})$,

(b) Let $Q = [\vec{u}_1 \ \vec{u}_2]$ be the matrix whose columns are \vec{u}_1 and \vec{u}_2 . Compute $QQ^T\vec{w}$. What do you notice?