7.4 worksheet

Recall from last time:

1. Consider a Markov chain (think: block party) where there are two nodes (houses). At each time interval, 70% of the people at node 1 go to node 2, and all the people from node 2 go to node 1. Suppose we start with 120 people at node 1 and 50 people at node 2. How many people will there be at each node after a really really long time?

Some closely related questions:

2. Let $A = \begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix}$, and let $\vec{x}_0 = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$. Find a "closed formula" for $A^t \vec{x}_0$.

3. Let $A = \begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix}$. Find a closed formula for A^t. (Hint: diagonalize A)

7.5 worksheet

- A *complex number* is a number of form ____________________, where ____________________, and $i =$ __________. The set of all complex numbers is denoted __________

- Addition, multiplication, division all work the same as usual:
  - $(3 + (\sqrt{2})i) + (5 - i) =$ __________
  - $(2 - 5i) - (1 - 5i) =$ __________
  - $(1 + 3i)(1 - 3i) =$ __________

- With complex numbers, there's another important operation called *complex conjugation*:

  For instance:
  - $\overline{3 - i} =$
  - $\overline{5} =$

- One reason conjugation is nice: simplifying fracationsc

- Big important fact (fundamental theorem of algebra): if $f(x)$ is any ____________________, then, using complex numbers, we can:

  e.g. $f(x) = x^4 - 9x^3 + 30x^2 - 42x + 20 = (x - 1)(x - 2)(x^2 - 6x + 10) =$

- Another important fact: if $f(x)$ is a polynomial with ____________________ and z is a root of $f(x)$, then

- E.g. find all Eigenvalues (real and complex!) of the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -4 \\ 0 & 4 & 3 \end{bmatrix}$$

- What are the eigenvectors?

- **Fact**: The eigenvalues and eigenvectors of a matrix with real entries come in ____________________ pairs

- Exercise: find the eigenvalues and eigenvectors of $\begin{bmatrix} 11 & -15 \\ 6 & -7 \end{bmatrix}$.