Final: focus on ch. 7, 8.

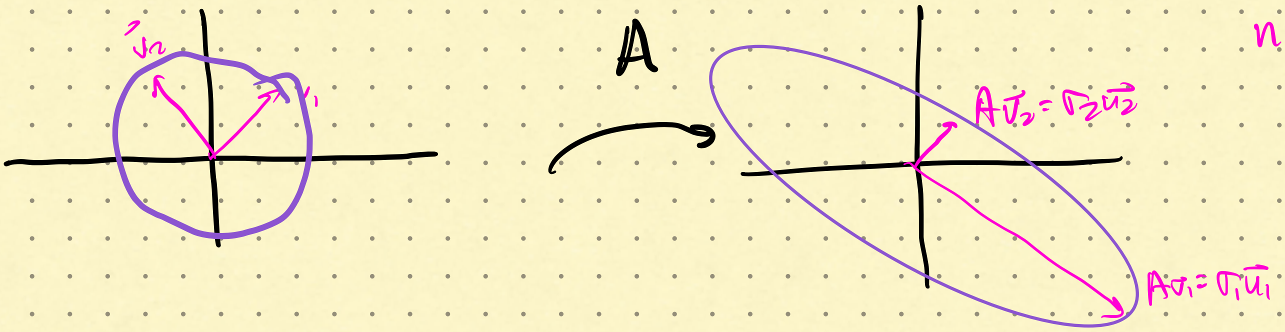
Agenda: lecture, then wksht.

Topic: Information you can see from SVD

Recall:

$$\underset{\substack{m\times n\\ \text{matrix}}}{A} = \underset{\substack{m\times m\\ \text{orthogonal}}}{U}\ \underset{\substack{m\times n\\ \text{"almost"}\\ \text{diagonal}}}{\Sigma}\ \underset{n\times n \text{ ortho}}{V^T} = [\vec{u}_1, \dots, \vec{u}_m] \underbrace{\begin{bmatrix} \sigma_1 \dots \sigma_n \\ \hline 0 \end{bmatrix}}_{n \leq m} \begin{bmatrix} - \vec{v}_1^T - \\ \vdots \\ - \vec{v}_n^T - \end{bmatrix}$$

$$\text{or} = [\vec{u}_1 \dots \vec{u}_m] \underbrace{\left[\begin{matrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_m \end{matrix} \middle| 0 \right]}_{n \geq m} \begin{bmatrix} - \vec{v}_1^T - \\ \vdots \\ - \vec{v}_n^T - \end{bmatrix}$$



Another interpretation:

SVD is equiv to:

$$\underset{m\times n}{A} = \sigma_1 \underbrace{\vec{u}_1 \vec{v}_1^T} + \sigma_2 \vec{u}_2 \vec{v}_2^T + \dots + \sigma_r \vec{u}_r \vec{v}_r \qquad (r = \min(m,n))$$

$$\underset{m\times 1}{[\ ]}\ \underset{1\times n}{[\qquad]} = \underset{m}{\left[\qquad\right]_n}$$

$\vec{u}_i \vec{v}_i^T (\vec{x}) = \vec{u}_1 (\vec{v}_i \cdot \vec{x})$

$\in \text{span}\{\vec{u}_i\}$.

$u_i v_i^T$ are particularly easy to work with:

$\text{im}(u_i \vec{v}_i^T) = \text{span}\{\vec{u}_i\}$, $\quad \ker(\vec{u}_i \vec{v}_i^T) = \text{span}\{\vec{v}_1, \dots, \vec{v}_{i-1}, \vec{v}_{i+1}, \dots, \vec{v}_n\}$

$$A = \sigma_1 \underbrace{\vec{u}_1 \vec{v}_1^T} + \dots + \sigma_r \vec{u}_r \vec{v}_r^T$$

Ingredients: Whole Grain Corn, Sugar, Corn Meal, Corn Syrup, Cocoa Processed with Alkali, Canola Oil, Fructose, Salt, Caramel Color, Refiner's Syrup, Baking Soda, Natural Flavor.

This perspective is useful in "dimensionality reduction", data compression etc.

e.g. if σ look like $\sigma_1 = 10$, $\sigma_2 = 9$, $\sigma_3 = 0.01$, $\sigma_4 = 0.005$

$$\Rightarrow A \approx \sigma_1 \vec{u}_1 \vec{v}_1^T + \sigma_2 \vec{u}_2 \vec{v}_2^T$$