Math 1060 Final Exam Review Sheet
December 2, 2014

The following review sheet is not exhaustive. It is meant to guide you in your finals studying by showing you which topics to focus on, as well as providing a diagnostic test for your understanding of the material. That means, completing this review sheet is a great first step to studying for the exam, but I recommend you work on other problems as well. Further, this review sheet reflects the material that I think is most important for everyone to know.

Simple harmonic motion and linear/rotational speed will not be on the final exam.

Section 4.1

1. Convert the following from radians to degrees:

   (a) $\dfrac{\pi}{12}$

   > **Solution:** $15°$

   (b) $\dfrac{-3\pi}{2}$

   > **Solution:** $-270°$

2. Find the length of the arc intercepted on a circle of radius 40 by an angle $\theta = \pi/5$ radians

   > **Solution:** 8π

3. Find the length of the arc intercepted on a circle of radius 40 by an angle $\theta = 45°$ degrees

   > **Solution:** 10π

Section 4.2

4. True or False: write "true" if the statement is true, and "false" otherwise.

   (a) $\sin\left(\dfrac{\pi}{3}\right) > \sin\left(\dfrac{\pi}{6}\right)$

   > **Solution:** True

   (b) $\sin\left(\dfrac{5\pi}{4}\right) = -\cos\left(\dfrac{5\pi}{4}\right)$

   > **Solution:** False

   (c) $\cos\left(\pi/2\right) = 0$

   > **Solution:** True

(d) $\sin\left(\dfrac{2\pi}{3}\right) > \cos\left(\dfrac{2\pi}{3}\right)$

> **Solution:** True

(e) sin is an odd function

> **Solution:** True

5. Let θ be some angle with $\cos(\theta) = 1/3$. What's $\cos(\theta + \pi)$?
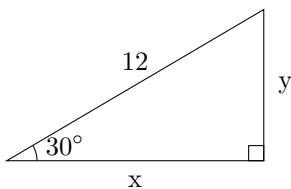
> **Solution:** $-\dfrac{1}{3}$

Section 4.3

6. Suppose θ is some angle so that $\sec(\theta) = \dfrac{5}{4}$ and $\sin\theta < 0$. Find all six trig functions of θ.

> **Solution:**
> $$\cos\theta = 4/5$$
> $$\sin\theta = -3/5$$
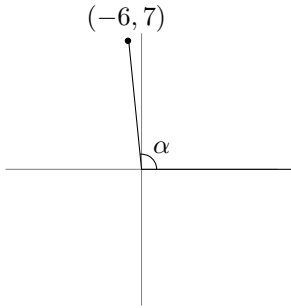> $$\tan\theta = -3/4$$
> $$\cot\theta = -4/3$$
> $$\csc\theta = -5/3$$

7. Determine x and y in the diagram below.

> **Solution:** $x = 6\sqrt{3}$, $y = 6$

Section 4.4

8. Find the six trig functions of α, where α is the angle in the diagram below

$(-6, 7)$

α

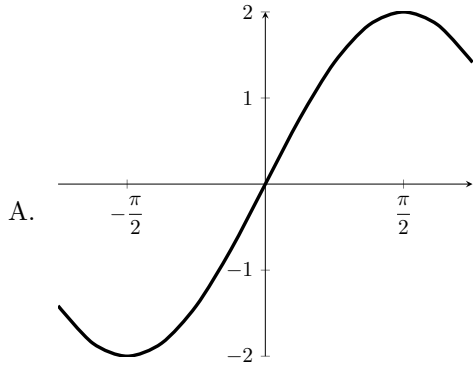**Solution:**

$$\sin \alpha = \frac{7}{\sqrt{85}}$$

$$\cos \alpha = -\frac{6}{\sqrt{85}}$$

$$\tan \alpha = -\frac{7}{6}$$

$$\cot \alpha = -\frac{6}{7}$$

$$\sec \alpha = -\frac{\sqrt{85}}{6}$$

$$\csc \alpha = \frac{\sqrt{85}}{7}$$

Section 4.5

9. Which of the following is a graph of $\sin(2x)$?

A.

B.

C.
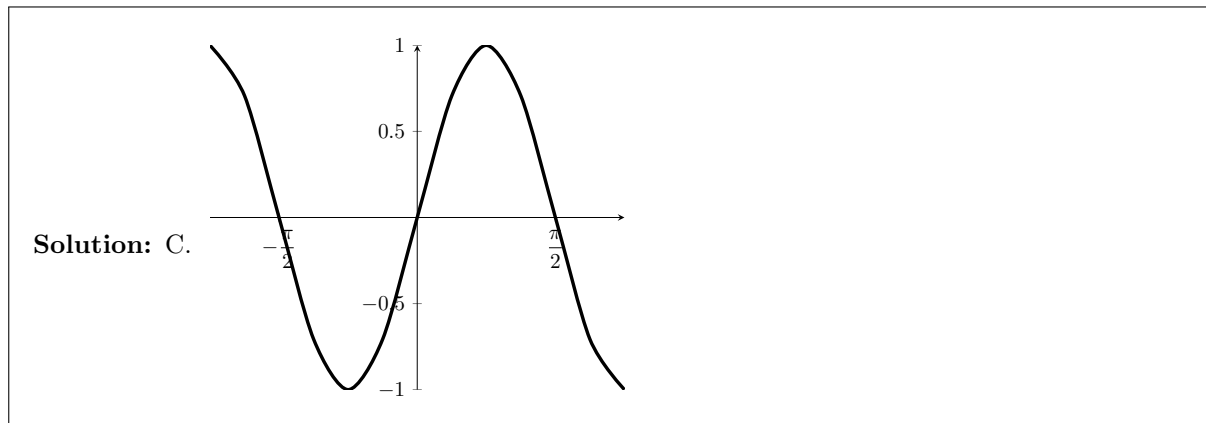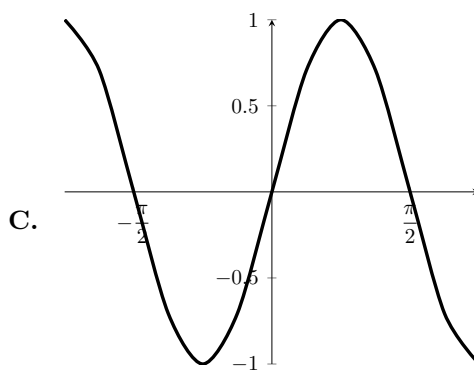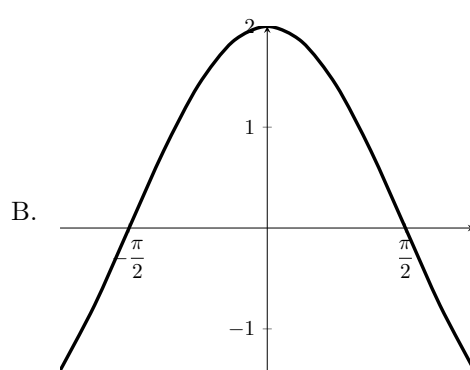
Solution: C.

10. Which of the following is a graph of $\cos\left(x - \dfrac{\pi}{4}\right)$?

A.

B.

C.
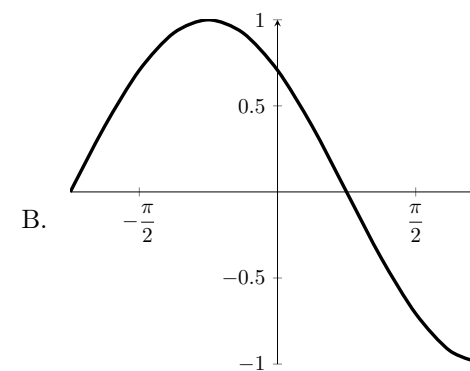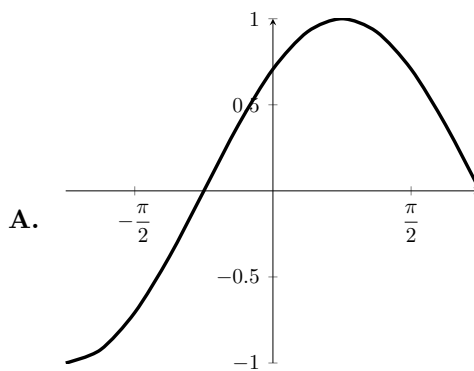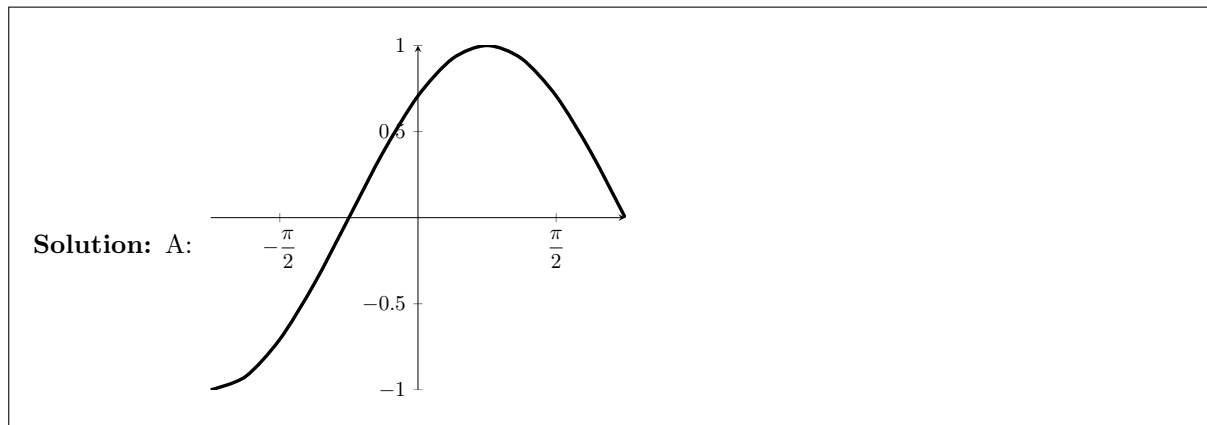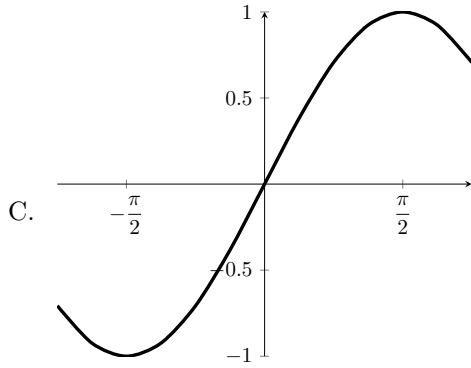
Solution: A:

Section 4.6

11. Plot $\tan x$ from $x = -\pi$ to $x = \pi$. Clearly label the asymptotes. Clearly label two points on the graph as well, one of which is nonzero.

> **Solution:** Check wolfram alpha.

Section 4.7

12. Write the domains and ranges of the following three functions:

   (a) arcsin

   > **Solution:** Domain: $[-1, 1]$, Range: $[-\pi/2, \pi/2]$

   (b) arccos

   > **Solution:** Domain: $[-1.1]$, Range: $[0, \pi]$

   (c) arctan

> **Solution:** Domain: $(-\infty, \infty)$, range: $(-\pi/2, \pi/2)$ (NOT $[-\pi/2, \pi/2]$)

13. Find the following, if possible. Note: it's only possible to evaluate $f(x)$ if x is in the *domain* of f.

(a) $\arcsin(-\sqrt{3}/2)$

> **Solution:** $-\dfrac{\pi}{3}$

(b) $\arccos(1/2)$

> **Solution:** $\dfrac{\pi}{3}$

(c) $\arctan(\sqrt{3})$

> **Solution:** $\dfrac{\pi}{3}$

(d) $\arccos(-\pi)$

> **Solution:** Can't evaluate $\arccos(-\pi)$

14. Find an algebraic expression for each of the following:

(a) $\sin\left(\arccos(x^2)\right)$
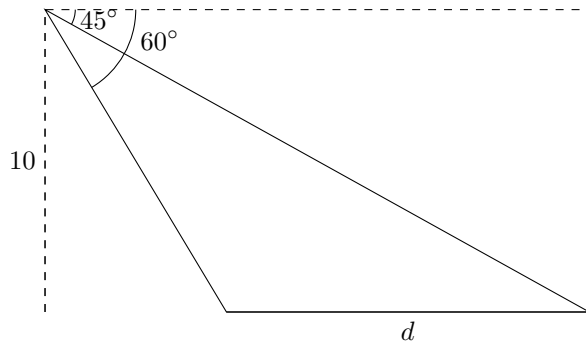
> **Solution:** $\sqrt{1 - x^4}$

(b) $\sec\left(\arcsin(x)\right)$

> **Solution:** $\dfrac{1}{\sqrt{1 - x^2}}$

(c) $\tan\left(\arccos(1/x)\right)$

> **Solution:** $\sqrt{x^2 - 1}$

Section 4.8

15. Solve for d:

> **Solution:** $10 - 10/\sqrt{3}$

Section 5.1/5.2

16. Add and simplify the following:

    (a) $\dfrac{1}{\sec x + 1} - \dfrac{1}{\sec x - 1}$

    > **Solution:** There are many answers. One is $2\sec x \cot^2 x$

    (b) $\tan x - \dfrac{\sec^2 x}{\tan x}$

    > **Solution:** There are many answers. One is $-\cot x$

    (c) $\dfrac{\sin x}{1 + \sin x} + \dfrac{\cos x}{\sin x}$

    > **Solution:** There are many answers. One is $\dfrac{\sin^2 x + \cos x + \cos x \sin x}{\sin x(1 + \sin x)}$

17. Factor the following expressions:

    (a) $\tan^2 x - \tan^2 x \sin^2 x$

    > **Solution:** $\tan^2 x \left(1 - \sin^2 x\right)$

    (b) $\sin^2 x \sec^2 x - \sin^2 x$

    > **Solution:** $\sin^2 x \left(\sec^2 x - 1\right)$

    (c) $\cot^2 + \csc x - 1$

> **Solution:** $(\csc x + 2)(\csc x - 1)$

18. (a) Show that $\cot^2 x \left(\sec^2 x - 1\right) = 1$

> **Solution:** Use the pythagorian identity and the reciprical identity

(b) Show that $(1 + \sin x)(1 - \sin x) = \cos^2 x$

> **Solution:** Multiply and use the pythagorean identity

Section 5.3

19. Solve each of the following equations for x
    (a) $\sin^2 x - 1/4 = 0$

> **Solution:** $x = \dfrac{\pi}{6} + 2n\pi,\ x = 5\pi/6 + 2n\pi,\ x = 7\pi/6 + 2n\pi,\ x = 11\pi/6 + 2n\pi.$ Equivalently,
> $x = \dfrac{\pi}{6} + n\pi,\ x = \dfrac{5\pi}{6} + n\pi.$

(b) $\cos^2 x - \cos x + 1/4 = 0$

> **Solution:** $x = \dfrac{\pi}{3} + 2n\pi,\ x = \dfrac{5\pi}{3} + n\pi$

(c) $\tan^2 x + (\sqrt{3} + 1)\tan x + \sqrt{3} = 0$

> **Solution:** $x = \dfrac{3\pi}{4} + n\pi,\ x = \dfrac{2\pi}{3} + n\pi$

Section 5.4

20. Find $\sin(\alpha + \beta)$ using the table of values below:

angle	sin	cos
α	0.2	$\sqrt{0.96}$
β	$\sqrt{0.91}$	0.3

> **Solution:** $0.06 + \sqrt{0.91 \cdot 0.96}$

Section 5.5

21. Suppose $\sin(\theta) = 0.6$ and $\cos(\theta) < 0$. Find $\sin(2\theta)$.

Solution: -0.96

22. Suppose $\sin^2(\theta) = 0.4$. What's $\cos(2\theta)$?

Solution: 0.2

Section 6.1/6.2

23. If I have a triangle with $a = 10$, $b = 15$, and $B = 30°$, what can you say about A?

Solution: $\sin A = 1/3$

24. If I have a triangle with $a = 15$, $b = 12$, and $C = 30°$, what can you say about c?

Solution: $c = \sqrt{15^2 + 12^2 - 180\sqrt{3}}$

Section 6.3/6.4

25. Let $v = \langle 1, 3 \rangle$, $u = \langle -1, 4 \rangle$.
    (a) Find $||v||$.

    Solution: $\sqrt{10}$

    (b) Find $u \cdot v$.

    Solution: 11

    (c) Find the projection of v onto u, i.e. $\text{proj}_u v$.

    Solution: $\langle \dfrac{-11}{17}, \dfrac{44}{17} \rangle$

Section 6.5

26. Write the complex number $2 + 2i$ in trig form (i.e. $r(\cos\theta + i\sin\theta)$).

Solution: $2\sqrt{2}\left(\cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4}\right)$

27. What's $\left(\dfrac{-1+i}{\sqrt{2}}\right)^{100}$?

> **Solution:** -1

Section 10.7/10.8

28. Convert $y = \pi x$ from rectangular to polar form

> **Solution:** $\tan\theta = \pi$

29. Graph the following polar equations:
    (a) $r = 2$
    (b) $r = \theta$
    (c) $r = \cos\theta$
    (d) $\theta = -\pi/4$
    (e) $r = \sin(3(\theta - \pi/6))$

> **Solution:** Check wolfram alpha.